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A Weighted-Logic Representation of C-Revising Ordinal Conditional Functions

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The problem of belief change is considered as a major issue in managing the dynamics of an information system. It consists in modifying an uncertainty distribution, representing agents' beliefs, in the light of a new information. In this paper, we focus on the so-called multiple iterated belief revision or C-revision, proposed for conditioning or revising uncertain distributions under uncertain inputs. Uncertainty distributions are represented in terms of ordinal conditional functions. We will use prioritized or weighted knowledge bases as a compact representation of uncertainty distributions. The input information leading to a revision of an uncertainty distribution is also represented by a set of consistent weighted formulas. This paper shows that C-revision, defined at a semantic level using ordinal conditional functions, has a very natural representation using weighted knowledge bases. We propose simple syntactic methods for revising weighted knowledge bases, that are semantically meaningful in the frameworks of possibility theory and ordinal conditional functions. In particular, we show that the space complexity of the proposed syntactic C-revision is linear with respect to the size of initial weighted knowledge bases.

Keywords: Uncertainty distributions; ordinal conditional functions; possibility theory; belief revision; weighted knowledge bases.

1. Introduction

Possibility theory and ordinal conditional functions (OCF) are uncertainty theories that offer convenient tools for handling uncertain or prioritized information. A crucial issue in these uncertainty theories is how to manage the dynamic of information systems. This is known as a belief revision[1–3] problem (or a conditioning problem) which is an important field of research in artificial intelligence and knowledge representation areas. It consists in defining processes for modifying initial beliefs in the light of new information, generally considered as fully reliable.

An example of application where belief revision is useful is the one of accessing to sensitive data or personal data that need to be protected. In such applications, It is crucial to equip information systems with mechanisms that control and restrict access to sensitive data to authorized users only. Access control is an essential component for the protection of information systems. Different types of access control exist in the literature such as Role-based Access Control (RBAC),[4] Organization-based access control (OrBAC)[5] and Attribute-Based Administration of Role-Based Access Control (AARBAC).[6,7] Current access control models have not the ability to provide explanations on the existing access control algorithms and/or results. When explanations are queried by administrative users, then there is often a need to revise security policies. For instance, assume that a unit director does not have access to a file o. Thanks to explanations, security policies may then need to be revised. The problem is difficult, because it is not enough to fix this aspect on an ad hoc basis (for instance by just adding an explicit permission for the unit director to access the object o), but rather to ensure that such situation cannot happen again in similar circumstances. Explainability may lead in this case to the proposal of minimal revision mechanisms of evolving security policies.

Three main components are needed to define a revision process. The first one concerns the representation of current beliefs or, more generally, of epistemic states. In the context of propositional logic, epistemic states are described by a set of consistent beliefs, a closed set of propositional formulas, or by a total pre-order over a set of propositional logic formulas. In this paper, at the semantic level, we will use ordinal conditional functions OCF[8,9] (or possibility theory) to represent epistemic states. Ordinal conditional functions[8,9] originally use classes of ordinals but are often defined using the integer scale. An ordinal conditional function, denoted by κ, is an uncertainty distribution where each element ω of the universe of discourse Ω (here a set of propositional logic interpretations) is associated with a positive integer number. The uncertainty degree $\kappa(\omega)$ is often interpreted as a degree of surprise that the interpretation ω is the real world. These uncertainty degrees may represent degrees of surprise or may simply encode an ordering on available information. Examples of belief revision methods that use ordinal conditional functions to represent epistemic states are conditionalisation,[8] C-revision (considered in this paper) and the so-called transmutations.[10]

At the syntactic level, uncertainty distributions (ordinal conditional functions or possibility distributions) are compactly represented using weighted logic bases. They are sets of pairs of the form (ϕ_i, α_i) where ϕ_i is a propositional logic formula and α_i is the uncertainty degree associated with ϕ_i.

Weighted knowledge bases have been intensively used in the literature for handling uncertainty such as in a possibilistic logic framework[11–13] or for handling inconsistency.[2,14–16]

Examples of weighted logics are: min-based and product-based possibilistic logic knowledge bases[11,17] that use the unit interval [0,1] to encode uncertainty degrees, and penalty logic weighted bases[18–20] that use the integer scale to encode the weights associated with formulas. In this paper, we more use penalty logic (or product-based possibilistic logic) where α_i's are positive integers.

The second component needed to define a revision process, concerns the representation of the new information. Typically, in belief revision, the new information is encoded by a formula of propositional logic. In some belief revision approaches, a new information can correspond to uncertain observations which may represent a partition over a set of interpretations as in Refs. 21–23. In some approaches, the input information is simply the whole epistemic states as in Ref. 24. In this paper, the new information will be represented by a consistent set of weighted propositional logic formulas.

The last element concerns the definition of the revision operator (or conditioning) itself, denoted by $\star$. In ordinal conditional functions framework, revision operations with uncertain input have been studied by Spohn[8] who has shown their close relationship with Jeffrey's rule[21] of revision in probability theory. Possibilistic counterparts to the revision by uncertain inputs have been discussed in Refs. 22, 23. In this paper, we will use the so-called C-revision, proposed in Ref. 25. This revision operator $(\star)$ takes as input an ordinal conditional function κ, a consistent set of weighted formulas $\mathcal{S}$, and produces a new ordinal conditional function $\kappa \star \mathcal{S}$. The revision operation takes into account the fact that formulas of the input $\mathcal{S}$ are issued from different and independent sources.

The main contribution of this paper consists in defining a syntactic representation of $\kappa \star \mathcal{S}$ using penalty logic. In this weighted logic, knowledge bases are sets of pairs (ϕ_i, α_i) where ϕ_i's are propositional logic formulas and α_i's are positive integers. The weight α_i is often interpreted as a degree of surprise or a price to pay if the propositional formula ϕ_i is not satisfied.

We show that C-revision, which is only defined at the semantic level using ordinal conditional functions, has a very natural counterpart in penalty logic, defined at the syntactic level using weighted (penalty logic-based) knowledge bases. We provide the syntactic counterpart of each step of the C-revision operation. One of the main advantage of our syntactic representation of C-revision is the space complexity: it is linear with respect to sizes of the initial weighted base and input information.

This paper is a revised and extended version of the conference paper.[26]

The rest of this paper is organized as follows. Section 2 gives a refresher on ordinal conditional functions and weighted knowledge bases. Sections 3 and 4 present the problems of belief revision and multiple iterated belief C-revision respectively. Section 5 gives the encoding of each step of the C-revision using weighted knowledge bases. Finally, Section 6 concludes the paper.

2. Ordinal Conditional Functions and Weighted Knowledge Bases

This section is divided into two subsections in which we first give a brief refresher on ordinal conditional functions (OCFs) (for more details see Ref. 8, 9) and then we present a weighted logic called penalty logic (for more details see Ref. 19).

2.1. *Ordinal conditional functions*

Let $\mathcal{L}$ be a propositional language based on a finite set of propositional variables. $\phi, \psi, \ldots$, etc. represent formulas of $\mathcal{L}$. $\perp$ and $\top$ represent the contradictory and the tautology formulas respectively. The set of propositional logic interpretations is represented by Ω. An interpretation of Ω is denoted by ω.

An ordinal conditional function, or simply an OCF distribution (we also use the term kappa function),[8,9] can be simply viewed as a function that assigns to each interpretation ω of Ω an integer, denoted by $\kappa(\omega)$. $\kappa(\omega)$ represents the degree of surprise of having the interpretation ω as being the real world. $\kappa(\omega) = 0$ means that nothing prevents ω for being the real world. The expression $\kappa(\omega) = 1$ means that the interpretation ω is somewhat surprising to be the real world. $\kappa(\omega) = +\infty$ simply means that it is impossible for ω to be the real world. The uncertainty degrees may also viewed as qualitative or abstract probabilities (e.g., Ref. 27) or infinitesimal probabilities.[28,29]

Example 1. Suppose that we are interested in encoding our beliefs regarding the amenities and facilities offered by a hotel in Paris' downtown.

Let c be a propositional symbol to express the fact that a hotel has a kitchen in the room. Let s be a propositional symbol to express that a hotel has a swimming pool.

Assume that available beliefs are expressed by the following ordinal conditional function κ given in Table 1.

Table 1. An example of ordinal conditional function.

ω			$\kappa(\omega)$
$\neg c$	$\wedge$	$\neg s$	0
c	$\wedge$	$\neg s$	1
$\neg c$	$\wedge$	s	1
c	$\wedge$	s	$+\infty$

The ordinal conditional function, given above, first represents the fact that having none of the two amenities is the normal situation. It also expresses that it is somewhat surprising to have one of the two amenities offered by the hotel. Lastly it is impossible that a hotel in Paris' downtown offers both amenities.

An ordinal conditional function κ defined on a set of interpretations Ω can be extended to formulas of the propositional language.

Given an OCF distribution κ defined on Ω, the weight of a propositional logic formula ψ, also denoted by $\kappa(\psi)$, is equal to the minimal weight of interpretations that satisfy ψ, namely:

$$\kappa(\psi) = \min\{\kappa(\omega), \omega \in \Omega, \omega \models \psi\}$$

with by convention $\kappa(\bot) = +\infty$.

In Example 1, if $\psi = c \vee s$ then:

$$\begin{aligned}\kappa(\psi) &= \min\{\kappa(\omega), \omega \in \Omega, \omega \models \psi\} \\ &= \min\{\kappa(c \wedge s), \kappa(\neg c \wedge s), \kappa(c \wedge \neg s)\} \\ &= 1\end{aligned}$$

2.2. *Weighted knowledge bases*

Weighted propositional logics are important frameworks for representing and reasoning with uncertain and inconsistent information in uncertainty-based frameworks.[30] Weighted knowledge bases are sets of weighted propositional logic formulas of the form (ϕ_i, α_i) where ϕ_i's are propositional logic formulas, and α_i's are positive numbers. Typically, α_i's are either positive integers or real numbers belonging to the unit interval [0, 1]. The interpretation of (ϕ_i, α_i) depends on the uncertainty framework considered. In possibility theory, (ϕ_i, α_i) means that the propositional formula φ_i is certain to at least a degree α_i. The weights α_i's may also represent degrees of surprise or prices to pay if φ_i's are not satisfied, as in penalty logic (considered here).[31,32] Penalty logic is a weighted logic which has been introduced in Ref. 18, and then developed in Refs. 19, 32.

More precisely, a penalty knowledge base $\mathcal{PK}$ is a finite set of pairs (ϕ_i, α_i) such that ϕ_i's are formulas of the propositional language $\mathcal{L}$ and α_i's are strictly positive integers[a]. The higher the weight α_i is, the more important the formula ϕ_i is.

In particular, if for some formula ϕ_i we have $\alpha_i = +\infty$ then ϕ_i is considered as an integrity constraint that should absolutely be satisfied. In the following, a weighted base $\mathcal{PK}$ is denoted by:

$$\mathcal{PK} = \{(\phi_i, \alpha_i), i = 1, \ldots, n\}.$$

[a] As we will see later, we will allow a negative integer to be only associated with the contradiction formula $\bot$.

If the weights of formulas in a penalty base $\mathcal{PK}$ are all equal to $+\infty$ (namely no formula of $\mathcal{PK}$ should be violated) penalty logic is then reduced to the propositional logic.

The following definition explains how to associate, to each weighted knowledge base, an ordinal conditional function over the set of interpretations.

Definition 1.[18] A weighted knowledge base $\mathcal{PK} = \{(\phi_i, \alpha_i) : i = 1, \ldots, n\}$ induces for each interpretation $\omega \in \Omega$ a weight, denoted $\kappa_{\mathcal{PK}}(\omega)$, which is equal to the sum of weights of formulas of $\mathcal{PK}$ that are not satisfied by ω. Namely $\forall \omega \in \Omega$:

$$\kappa_{\mathcal{PK}}(\omega) = \begin{cases} 0 & \text{if} \quad \forall (\phi_i, \alpha_i) \in \mathcal{PK}, \omega \models \phi_i\,, \\ \sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK}, \omega \not\models \phi_i\} & \text{otherwise} \end{cases}$$

In the following, $\kappa_{\mathcal{PK}}$ is simply called the ordinal conditional function OCF associated with $\mathcal{PK}$. The way the ordinal conditional function distributions are produced from penalty logic bases is very close to the way possibility distributions are produced from product-based possibilistic knowledge bases.[17]

One can easily check that if a formula ϕ appears several times in a weighted knowledge base $\mathcal{PK}$, we can replace all its occurrences by a single occurrence of the formula ϕ weighted by the sum of the weights associated with this formula.

Namely, if $(\phi, \alpha) \in \mathcal{PK}$ and $(\phi, \beta) \in \mathcal{PK}$ then $\mathcal{PK}$ is equivalent to $(\mathcal{PK} - \{(\phi, \alpha), (\phi, \beta)\} \cup \{(\phi, \alpha + \beta)\})$, in the sense that they induce a same ordinal conditional function.

Example 2. Let us consider a variant of Example 1 where we are also interested in encoding our beliefs on amenities offered by an hotel in Paris' downtown.

Let us focus on only three amenities: (i) there is a free parking or not, (ii) rooms have kitchen or not and (iii) there is a daily housekeeping or not.

These amenities are represented by the following three propositional symbols p, c and h respectively.

Assume that our beliefs are represented by the following ordinal conditional function given in Table 2.

Table 2. An example of an ordinal conditional function κ representing amenities of a hotel in Paris' downtown.

ω	p	c	h	$k(\omega)$
ω_0	0	0	1	0
ω_1	0	0	0	1
ω_2	0	1	1	1
ω_3	1	0	1	1
ω_4	0	1	0	2
ω_5	1	1	1	2
ω_6	1	0	0	7
ω_7	1	1	0	8

The ordinal conditional function, given in Table 2, expresses the fact that the normal situation is given by the interpretation ω_0. Namely the normal situation is to have a daily housekeeping and where the hotel neither offers a parking facility nor provides a kitchen in the hotel's rooms. Surprising situations are given by the interpretations ω_1, ω_2 or ω_3. Each of these three interpretations represent a deviation from the normal situation with respect to exactly one of the three amenities.

More surprising situations are given by the two interpretations ω_4 and ω_5. Both of these interpretations depart from the normal situation with respect to two amenities.

A very surprising situation is given by the interpretation ω_6. It corresponds to the situation where a Paris' downtown hotel is offering a parking facility without offering neither a daily housekeeping nor a kitchen in the rooms.

Lastly, the worst situation is given by the interpretation ω_7 which simply corresponds the opposite situation with respect to the normal situation.

3. Belief Revision in Uncertainty Frameworks

Belief revision was originally introduced by Alchourron, Gärdenfors and Makinson.[1,33] They proposed a set of axioms (called AGM postulates) to characterize the rationality of a revision process.

In Ref. 34 the authors have proposed an extension of the AGM model by adding postulates that manage the iterated revision. Jin and Thielscher[35] have also proposed a new postulate for iterated revision called *Independance* postulate. This later was generalized by Delgrande and jin[36] by introducing a new set of postulates that surpasses weakness of postulates proposed in Ref. 34.

3.1. *Revising uncertainty distributions*

In uncertainty theories, several works have been proposed for revising, or conditioning, ordinal conditional functions[37,38] and possibility distributions.[22,23]

In possibility theory, the revision of possibilistic knowledge bases has been proposed, for example in Refs. 22, 23, using the possibilistic counterpart of Jeffrey's rule.[21] Possibility theory has strong connections with ordinal conditional functions. In possibility theory, the uncertainty distribution, called a possibility distribution, is denoted by π. It assigns to each interpretation ω a positive real number in the unit interval $[0, 1]$. By convention $\pi(\omega) = 1$ means that ω is among the most normal (or preferred) situations while $\pi(\omega) = 0$ means that ω is impossible or excluded for being a possible solution. More generally, if $\pi(\omega) > \pi(\omega')$ then ω is considered as more preferred, or more consistent given available knowledge, than ω'. There are several compact (or syntactic) encoding of possibility distributions: possilistic knowledge bases, guaranteed-based weighted knowledge bases, possibilistic graphs, etc. The problem of belief revision has been more investigated in the context of possibility distributions (at the semantic level) and possibilistic knowledge bases (at the syntactic level).

The revision of, or conditioning, a possibility distribution π by an input composed of a set of weighted formulas $\mu = \{(\phi_i, a_i), i = 1, \ldots, n\}$ is defined as follows:

$$\forall(\phi_i, a_i) \in \mu, \forall\omega \vDash \phi_i, \pi(\omega|\mu) = a_i \otimes \pi(\omega|_{\otimes}\phi_i)$$

where $\otimes$ represents either the product operator or the min operator depending on the nature of the input and on the definition of conditioning used in the possibility theory. Note that, in Jeffrey's rule, as well as in its possibility theory counterpart, formulas of the input ϕ_i's $\in \mu$ represent a partition, namely

$$\bigvee\{\phi_i, \phi_i \in \mu\} \text{ is a tautology}$$

and

$$\forall i, \forall j, \phi_i \wedge \psi_j \text{ is a contradiction (with } i \neq j).$$

In the context of ordinal conditional functions, Spohn[8] has introduced different revision operators or conditioning concepts in the context of ordinal conditional functions. The so-called conditionalisation transforms an ordinal conditional function κ, into a new one, denoted by κ', in the presence of a new information (ϕ, i). First, Spohn introduced the notion of ϕ-part of κ as follows:

- the ϕ-*part* of κ is the conditioning by a non-weighted propositional formula ϕ defined by

$$\forall\omega \in \Omega, \kappa(\omega|\phi) = \kappa(\omega) - \kappa(\phi)\,.$$

- the (ϕ, i)-*conditionalization* of κ, say $\kappa(\omega|(\phi, i))$ is a conditioning operation by an uncertain input $\kappa'(\phi) = i$, defined by

$$\begin{aligned}\kappa(\omega|(\phi, i)) &= \kappa(\omega|\phi) \text{ if } \omega \vDash \phi\\ &= i + \kappa(\omega|\neg\phi) \text{ if } \omega \vDash \neg\phi\,.\end{aligned}$$

The following section presents a revision process that operates on ordinal conditional functions. This revision operation is called C-revision and provides another view on conditioning under a set of uncertain information.

3.2. *C-revision*

This section briefly presents the C-revision operation introduced in Refs. 25, 39. The C-revision operation considers that the input of the revision process is a set of consistent weighted formulas (and not a single formula) $\mathcal{S} = \{(\xi_1, \beta_1), \ldots, (\xi_n, \beta_n)\}$. We assume that the set of propositional formulas $\{\xi_1, \ldots, \xi_n\}$ is also consistent with fully certain formulas of the initial knowledge base; namely $\{\xi_1, \ldots, \xi_n\}$ is consistent with $\{\phi_i : (\phi_i, \alpha_i) \in \mathcal{PK}$ and $\alpha_i = +\infty\}$. Recall that formulas with $+\infty$ weights encode fully certain information and cannot be questionable in the light of new in formation.

The way the input is viewed in C-revision departs from the one used in Jeffrey's rule where the input formulas induce a partition over Ω, since ξ_i's are assumed here to be jointly consistent.

The C-revision is only defined at a semantic level for ordinal conditional functions. It takes as input an ordinal conditional function distribution κ, a consistent set of weighted formulas and produces a new ordinal conditional function distribution denoted by $\kappa \star \mathcal{S}$. The C-revision, even if it is defined in the context of belief revision, can be viewed as a form of conditioning under uncertain input.

Definition 2. Let κ be an ordinal conditional function distribution and let $\mathcal{S} = \{(\xi_1, \beta_1), \ldots, (\xi_n, \beta_n)\}$ be a consistent set of weighted formulas where weights are positive integers ($\beta_i \in \mathcal{N}^+$).

The new ordinal conditional function distribution $\kappa \star \mathcal{S}$, obtained after the C-revision of κ by $\mathcal{S}$ using $\star$, is defined as follows:

- $\forall \omega \in \Omega$,

$$\kappa \star \mathcal{S}(\omega) = \kappa(\omega) - \kappa(\xi_1 \wedge \cdots \wedge \xi_n) + \sum_{i=1, \omega \not\models \xi_i}^{n} \beta_i \,.$$

- Each β_i ($i \in \{1, \ldots, n\}$) satisfies the following condition:

$$\beta_i > \kappa(\xi_1 \wedge \cdots \wedge \xi_n) - \min_{\omega \not\models \xi_i} \{\kappa(\omega) + \sum_{j \neq i, \omega \not\models \xi_j}^{n} \beta_j\} \,.$$

One can easily check that if an interpretation ω is one of the best model of $\xi_1 \wedge \cdots \wedge \xi_n$ then $\kappa \star \mathcal{S}(\omega) = 0$. We say that an interpretation ω is a best model of $\xi_1 \wedge \cdots \wedge \xi_n$ with respect to κ if:

- ω is a model of $\xi_1 \wedge \cdots \wedge \xi_n$ and
- there is no other model ω' of $\xi_1 \wedge \cdots \wedge \xi_n$ such that $\kappa(\omega') < \kappa(\omega)$.

Therefore if ω is a model of $\xi_1 \wedge \cdots \wedge \xi_n$ then the expression $\sum_{i=1, \omega \not\models \xi_i}^{n} \beta_i$ is trivially equal to 0. Besides, since ω is one of the best model of $\xi_1 \wedge \cdots \wedge \xi_n$ then this also means that $\kappa(\omega) = \kappa(\xi_1 \wedge \cdots \wedge \xi_n)$. Hence, $\kappa \star \mathcal{S}(\omega) = \kappa(\omega) - \kappa(\xi_1 \wedge \cdots \wedge \xi_n) + \sum_{i=1, \omega \not\models \xi_i}^{n} \beta_i = 0$.

Besides counter-models of ξ_i's are shifted up by their associated weights β_i's as it is stated by the expression $\sum_{i=1, \omega \not\models \xi_i}^{n} \beta_i$ used in defining $\kappa \star \mathcal{S}$.

In Definition 2, the presence of the sum operator ($\sum$) in the definition of $\kappa \star \mathcal{S}$ reflects the fact that the formulas ξ_i may be issued from independent sources. Note that the use of the sum operator is fully in a spirit of penalty logic: the more formulas in $\mathcal{S}$ falsified by some interpretation ω the higher is the degree of $\kappa \star \mathcal{S}(\omega)$ and the more surprising will be the interpretation ω in the revised ordinal conditional function $\kappa \star \mathcal{S}$.

Lastly, the inequality constraints on β_i's given in Definition 2 ensures that $\kappa \star \mathcal{S}(\neg \xi_i) > 0$ for each (ξ_i, β_i) in $\mathcal{S}$.

It has been shown in Ref. 25 that multiple iterated C-revision satisfies the extended AGM postulates (as described in Ref. 25), and the two postulates (PC3) and (PC4) of Delgrande and jin.[36]

Table 3. The result of revising the ordinal conditional function given in Table 2 with the input $\mathcal{S} = \{(p, \alpha), ((h \wedge \neg c) \vee (\neg h \wedge c))\}$.

ω	p	c	h	$k \star \mathcal{S}(\omega)$
ω_0	0	0	1	$\alpha - 1$
ω_1	0	0	0	$\alpha+\beta$
ω_2	0	1	1	$\alpha+\beta$
ω_3	1	0	1	0
ω_4	0	1	0	$1+\alpha$
ω_5	1	1	1	$1+\beta$
ω_6	1	0	0	$6+\beta$
ω_7	1	1	0	7

It has also been argued in Ref. 25 that Jin's and Thielscher's (Ind) postulate[35] is not necessary for multiple iterated C-revision.

Example 3. Let us continue our example. More precisely le us consider again the ordinal conditional function given by Table 2.

Assume that we have the new information:

$$\mathcal{S} = \{(p, \alpha), ((h \wedge \neg c) \vee (\neg h \wedge c), \beta)\}\,.$$

This set of weighted formulas (encoding for instance observations regarding hotels in a new area of Paris) expresses a new information where hotels have parking facilities, provides either a daily housekeeping or (exclusively) a kitchen in rooms.

Let us revise the ordinal conditional function in Table 2 using Definition 2.

From Table 2, we have:

$$k(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))) = 1\,.$$

Revising the ordinal conditional function in Table 2 gives us the ordinal conditional function $k \star \mathcal{S}$ illustrated in Table 3:

Let us now specify the constraints that α and β should satisfy. Recall that $k(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))) = 1$. From Definition 2, we have:

$$\begin{aligned}\alpha &> \kappa(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))) - \min_{\omega \not\models p}\{\kappa(\omega) + \{\beta : if\ \omega \not\models ((h \wedge \neg c) \vee (\neg h \wedge c)\}\} \\ &> 1 - \min_{\omega \not\models p}\{\kappa(\omega) + \{\beta : if\ \omega \not\models ((h \wedge \neg c) \vee (\neg h \wedge c)\}\}\end{aligned}$$

which leads to:

$$\alpha > 0\,.$$

Similarly, one can check that:

$$\beta > 1 + \alpha\,.$$

In Ref. 40, the syntactic counterpart based on min-based possibilistic weighted bases has been proposed. The computation of the ordinal conditional function κ associated with $\mathcal{PK}$ is obtained by an equation similar to the one given in Definition 1 (the maximum operator is used instead of the sum operator). The major disadvantage of the syntactic representation proposed in Ref. 40 is that the obtained revised base is larger (exponential) than the original knowledge base.

The aim of the following section is to propose a compact and direct encoding of multiple C-revision using penalty logic with a linear space complexity.

4. Syntactic Computation of C-Revision

This section provides a syntactic computation of C-revision. More precisely, let $\mathcal{PK}$ be a weighted knowledge base; encoding initial beliefs. Let $\kappa_{\mathcal{PK}}$ be the ordinal conditional function associated with a knowledge base $\mathcal{PK}$ using Definition 1. Let $\mathcal{S} = \{(\xi_1, \beta_1), \ldots, (\xi_n, \beta_n)\}$ be a consistent set of weighted formulas representing new information. Lastly, let $\kappa_{\mathcal{PK}} \star \mathcal{S}$ be the result of revising $\kappa_{\mathcal{PK}}$ with the input $\mathcal{S}$.

Our aim is to compute a new weighted base $\mathcal{PK}1$ (from $\mathcal{PK}$ and $\mathcal{S}$) such that:

$$\forall \omega \in \Omega, \kappa_{\mathcal{PK}1}(\omega) = \kappa_{\mathcal{PK}} \star \mathcal{S}(\omega)\,,$$

where $\kappa_{\mathcal{PK}1}$ (resp $\kappa_{\mathcal{PK}}$) is the ordinal conditional function associated with $\mathcal{PK}1$ (resp $\mathcal{PK}$) given by Definition 1, and $\kappa_{\mathcal{PK}} \star \mathcal{S}$ is the C-revision of $\kappa_{\mathcal{PK}}$ by $\mathcal{S}$ using Definition 2.

Before presenting the main steps of computing $\kappa_{\mathcal{PK}1}$, let us first present the example of a weighted knowledge base which will be used in the rest of this paper.

Example 4. To illustrate the main steps of our syntactic method, we will consider the following weighted knowledge base:

$$\mathcal{PK} = \{(\neg p, 1), (\neg c, 1), (h, 1), (\neg p \vee h, 5)\}\,.$$

One can check that the ordinal conditional function $k_{\mathcal{PK}}$, given by Definition 1, is exactly the same as the one given in Table 2.

For instance, for ω_4 we have:

$$\begin{aligned} k_{\mathcal{PK}}(\omega_4) &= \textstyle\sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK}, \omega_4 \not\models \phi_i\} \\ &= 1(\omega_4 \not\models \neg c) + 1(\omega_4 \not\models h) \\ &= 2 \end{aligned}$$

Hence $\mathcal{PK}$ is indeed a compact representation of the ordinal conditional distribution given in Table 2.

In the above example, the degrees, associated with formulas of the knowledge base, represent the degrees of surprise if such associated formulas are not satisfied. Hence, the preferred interpretations are those that are models of knowledge bases (namely, $\omega_0 = \neg p \wedge \neg c \wedge h$). The second preferred interpretations are those that only falsify one formula of degree 1 from the weighted knowledge base. An example of such interpretation is $\omega_1 = p \wedge \neg c \wedge h$. The worst interpretation is the one that falsifies all formula of the knowledge base, namely $\omega_0 = p \wedge c \wedge \neg h$. Note that the major difference between penalty logic and min-based possibilitic logic concerns the way the degrees, associated with interpretations are defined. In penalty logic the degrees associated with falsified formulas are added to define the degree of an interpretation. In min-based possibilistic logic, only the highest falsified formula is considered for defining the degree of an interpretation.

4.1. *Main steps of the syntactic computation of C-revision*

Recall that our aim is to compute a new weighted base $\mathcal{PK}1$, from the initial weighted knowledge base $\mathcal{PK}$ and the input $\mathcal{S}$ such that: $\forall\omega, \kappa_{\mathcal{PK}1}(\omega) = \kappa_{\mathcal{PK}}\star\mathcal{S}(\omega)$.

From Definition 2, the revision operation C-revision is defined by:
$\forall\omega \in \Omega$,

$$\kappa_{\mathcal{PK}} \star \mathcal{S}(\omega) = \kappa_{\mathcal{PK}}(\omega) - \kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n) + \sum_{i=1,\omega \not\models u_i}^{n} \beta_i \,.$$

Thus, given the above equation regarding the definition of C-revision, the syntactic C-revision requires the following three main steps:

- A syntactic computation, from the weighted knowledge base $\mathcal{PK}$, of $\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n)$;
- A syntactic computation of the weighted base associated with the ordinal conditional function κ' defined by:

$$\forall\omega \in \Omega, \kappa'(\omega) = \kappa_{\mathcal{PK}}(\omega) + \sum_{i=1,\omega \not\models \xi_i}^{n} \beta_i;$$

- A syntactic computation of the weighted base associated with $\kappa_{\mathcal{PK}} \star \mathcal{S}$.

These three main steps are described in the following three subsections respectively.

4.2. *Syntactic computation of the input weight*

Let us start with the syntactic computation of $\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n)$. Recall first that:

$$\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n) = min\{\kappa_{\mathcal{PK}}(\omega) : \omega \in \Omega, \omega \models \xi_1 \wedge \cdots \wedge \xi_n\} \,.$$

To syntactically compute $\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n)$ for a given weighted knowledge base $\mathcal{PK}$, we need to consider the following notations:

- $\mathcal{PK}^\star$: is the propositional base obtained by only considering propositional formulas of $\mathcal{PK}$ without taking into account their weights, namely:

$$\mathcal{PK}^\star = \{\phi_i, (\phi_i, \alpha_i) \in \mathcal{PK}\} \,.$$

For instance, if

$$\mathcal{PK} = \{(\neg a, 2), (b, 7), (c \vee \neg b, 5), (a \vee c, +\infty)\}$$

then

$$\mathcal{PK}^\star = \{\neg a, b, c \vee \neg b, a \vee c\}\,.$$

- $SW(K)$: is a function that sums the weights of a sub-base K of $\mathcal{PK}$, namely:

$$SW(K) = \sum\{\alpha_i : (\phi_i, \alpha_i) \in K \text{ and } \alpha_i \neq +\infty\}\,.$$

For instance, if

$$\mathcal{PK} = \{(\neg a, 2), (b, 7), (c \vee \neg b, 5), (a \vee c, +\infty)\}$$

and

$$K = \{(b, 7), (c \vee \neg b, 5)\}$$

then

$$SW(K) = 7\,.$$

Then the syntactic computation of $\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n)$ is obtained using the following proposition:

Proposition 1. *Let:*

- $\mathcal{PK}$*: be a weighted knowledge base;*
- ψ*: be a consistent formula (here* $\psi = \xi_1 \wedge \cdots \wedge \xi_n$*);*
  We also assume that ψ *is consistent with fully certain formulas of* $\mathcal{PK}$*; namely* ψ *is consistent with* $\{\phi_i : (\phi_i, \alpha_i) \in \mathcal{PK} \text{ and } \alpha_i = +\infty\}$.
- A*: be a sub-set of* $\mathcal{PK}$ *such that:*
  — $\{(\phi_i, \alpha_i) : (\phi_i, \alpha_i) \in \mathcal{PK} \text{ and } \alpha_i = +\infty\} \subset A$.
  — $A^\star \wedge \psi$ *is consistent and*
  — $\nexists B \subseteq \mathcal{PK}$ *such that* $B^\star \wedge \psi$ *is consistent,* $\{(\phi_i, \alpha_i) : (\phi_i, \alpha_i) \in \mathcal{PK} \text{ and } \alpha_i = +\infty\} \subset B$ *and* $SW(B) > SW(A)$.

Then:

$$\begin{aligned}\kappa_{\mathcal{PK}}(\psi) &= \min\{\kappa_{\mathcal{PK}}(\omega), \omega \models \psi\}\\ &= SW(\mathcal{PK}) - SW(A)\,.\end{aligned}$$

Proof. Note first that, by assumption, the new input formula ψ is consistent with the set of fully certain formulas of the knowledge base; namely ψ is consistent with $\{\phi_i : (\phi_i, \alpha_i) \in \mathcal{PK} \textit{ and } \alpha_i = +\infty\}$. Hence, there exists at least a model of ψ with a weight different from $+\infty$. From Definition 1 we have:

$$\begin{aligned}\kappa_{\mathcal{PK}}(\psi) &= \min\{\kappa_{\mathcal{PK}}(\omega), \omega \models \psi\}\\ &= \min_{\omega \models \psi}\{\sum\{\alpha_i, \omega \models \psi \wedge \neg\phi_i\}\}\end{aligned}$$

$$= \min{}_{\omega} \models \psi \{SK(\mathcal{PK}) - \sum\{\alpha_i, \omega \models \phi_i \wedge \psi\}\}$$
$$= SW(\mathcal{PK}) - \max_{\omega \models \psi}\{\sum\{\alpha_i, \omega \models \phi_i \wedge \psi\}\}$$
$$= SW(\mathcal{PK}) - \max_{\omega \models \psi}\{SW(C), C \subseteq \mathcal{PK}, \omega \models C^\star \wedge \psi\}\}$$
$$= SW(\mathcal{PK}) - \max\{SW(C), C \subseteq \mathcal{PK}, C^\star \wedge \psi \text{ } consistent\}\}$$
$$= SW(\mathcal{PK}) - SW(A)\,.$$

(For the sake of the proof, $\sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK}\}$ is replaced by $\sum\{\alpha_i\}$.) □

Intuitively, in the above proposition the subbase A represents one of the maximal consistent subbase of $\mathcal{PK}$, where maximality is understood here with respect to the sum of the weights of the formulas present in A.

The computation of a sub-base A (and $SW(A)$) satisfying conditions given in the above proposition can be obtained using a polynomial number of calls to a Partial weighted Max-SAT test.

The weighted Max-SAT problem generalizes the SAT problem: given a set of clauses with non-negative integer weights on each clause, find an assignment of variables that maximizes the sum of the weights of the satisfied clauses. Its associated decision problem has as input i) a set X of clausal formulas with non-negative integer weights on each clause, and i) a positive integer number k. The decision question is: is there an instantiation of propositional variables of the language (or an interpretation) such that the sum of weights of satisfied clauses in X is greater or equal to k? The decision problem MaxSat is NP-complete.[41]

The decision problem Partial weighted Max-SAT is a variant of weighted MAX-SAT that takes into account fully certain formulas (namely formulas having $+\infty$ weights). Note that if the knowledge base $\mathcal{PK}$ is empty or only composed of contradictory formulas, then the weighted Max-SAT decision problem returns a positive answer only for the integer $k = 0$. In this case, only the empty set is a solution of the subbase A (as described in the above Proposition 1). Now, if the conjunction of clauses in $\mathcal{PK}$ is consistent, then the weighted Max-SAT decision problem returns a positive answer for all integers k in $\{0, \ldots, \sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK} \text{ } and \text{ } \alpha_i \neq +\infty\}\}$. And hence in this case, $A = \mathcal{PK}$ is the only solution of the subbase A given in Proposition 1.

More generally, to compute a subbase A (as described in the above Proposition 1), it is enough to apply a dichotomic search over the set of all possible weights in $\{0, \ldots, \sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK} \text{ } and \text{ } \alpha_i \neq +\infty\}\}$ and then for each weight γ of this set, a call to Partial weighted Max-SAT decision problem is achieved with a weight γ. One can check that the size of $\{0, \ldots, \sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK} \text{ } and \text{ } \alpha_i \neq +\infty\}\}$ is bounded by $2^{|\mathcal{PK}|}$ where $\mid \mathcal{PK} \mid$ is the number of weighted clauses in $\mathcal{PK}$. Since the dichotomic search over a set of size m has a complexity of $\mathcal{O}(m)$, the computational complexity of computing $SW(A)$, where A is a subbase described in Proposition 1, is equal to $\mathcal{O}(|\ \mathcal{PK}\ |)$ calls to Partial weighted Max-SAT decision problem.

Example 5. Let us continue our example with:

$$\mathcal{PK} = \{(\neg p, 1), (\neg c, 1), (h, 1), (\neg p \vee h, 5)\}$$

and

$$\mathcal{S} = \{(p, \alpha), ((h \wedge \neg c) \vee (\neg h \wedge c), \beta)\}\,.$$

An example of a sub-set A such that:

- $A^\star \wedge p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))$ is consistent, and
- $\nexists B \subseteq \mathcal{PK}, B$ consistent with $(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))$ *and* $SW(B) > SW(A))$,

is $A = \{(\neg c, 1), (h, 1), (\neg p \vee h, 5)\}$. Indeed, first note that $A^\star = \{\neg c, h, \neg p \vee h\}$ is consistent with $(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c)))$ and that $SW(A) = 7$. One can also check that there is no $B \subseteq \mathcal{PK}$ such that $B^\star$ is consistent and $SW(B) > SW(A)$. Indeed, the only way to satisfy $SW(B) > SW(A)$ is to set $B = \mathcal{PK}$ (and hence $SW(B) = 8$). But this is impossible since $\mathcal{PK}^\star$ is inconsistent with $(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c)))$.

Using Proposition 1:

$$\kappa_{\mathcal{PK}}(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))) = SW(\mathcal{PK}) - SW(A) = 1\,.$$

Besides, one can also verify from Table 2 that:

$$\kappa_{\mathcal{PK}}(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))) = 1.$$

Hence, computing the initial degree associated with the input formula (namely $(p \wedge ((h \wedge \neg c) \vee (\neg h \wedge c))))$ can be either obtained semantically using the ordinal conditional function (given in Table 2) or obtained syntactically from its associated weighted knowledge base, thanks to Proposition 1.

Next section is devoted to the integration of the input in the weighted knowledge base.

4.3. *The integration of the input $\mathcal{S}$ in the weighted base*

This subsection provides the syntactic counterpart of the result of integrating the new information into the weighted knowledge base. Note that in the definition of $\kappa_{\mathcal{PK}} \star \mathcal{S}$, each interpretation ω is shifted up by the sum of the weights of formulas in $\mathcal{S}$ that are falsified by the interpretation ω. The way interpretations are shifted up in the presence of the input information $\mathcal{S}$ is exactly the same as the way interpretations are evaluated with respect to the weighted knowledge base. Therefore the syntactic computation of $\kappa_{\mathcal{PK}'}(\omega) = \kappa_{\mathcal{PK}}(\omega) + \sum_{(\xi_i,\beta_i)\in\mathcal{S},\ \omega\not\models\xi_i} \beta_i$ is immediate as indicated in Proposition 2.

Proposition 2. *Let* $\mathcal{PK} = \{(\phi_i, \alpha_i)\colon\ i = 1, \ldots, n\}$ *be a weighted knowledge base, and* $\kappa_{\mathcal{PK}}$ *be its associated distribution using Definition 1. Let* $\mathcal{S} =$

Table 4. The ordinal conditional distribution associated with $\mathcal{PK}'$ after integrating the input.

ω	p	c	h	$k_{\mathcal{PK}'}(\omega)$
ω_0	0	0	1	α
ω_1	0	0	0	$1+\alpha+\beta$
ω_2	0	1	1	$1+\alpha+\beta$
ω_3	1	0	1	1
ω_4	0	1	0	$2+\alpha$
ω_5	1	1	1	$1+2+\beta$
ω_6	1	0	0	$7+\beta$
ω_7	1	1	0	8

$\{(\xi_1, \beta_1), \ldots, (\xi_n, \beta_n)\}$ *be the input. The syntactic counterpart of:*

$$\kappa_{\mathcal{PK}'}(\omega) = \kappa_{\mathcal{PK}}(\omega) + \sum_{(\xi_i,\beta_i)\in\mathcal{S},\ \omega\not\models\xi_i} \beta_i$$

is:

$$\mathcal{PK}' = \mathcal{PK} \cup \{(\xi_i, \beta_i), i = 1, \ldots, n\} ..$$

The proof is immediate. It is enough to notice that $\forall i \in \{1, \ldots, n\}$ and $\forall \omega \in \Omega$,

$$\kappa_{\mathcal{PK}\cup\{(\xi_i,\beta_i)\}}(\omega) = \begin{cases} \kappa_{\mathcal{PK}}(\omega) + \beta_i, & \text{if } \omega \not\models \xi_i\,. \\ \kappa_{\mathcal{PK}}(\omega), & \text{otherwise} \end{cases}$$

Hence, it is enough to iterate the above equation over all weighted formulas in $\mathcal{S}$ to get the full proof of the proposition. Clearly, given a weighted knowledge base $\mathcal{PK}$ and a single formula $\{(\xi_i, \beta_i)\}$ computing the new knowledge base associated to $\kappa_{\mathcal{PK}\cup\{(\xi_i,\beta_i)\}}(.)$ is in $\mathcal{O}(1)$. Besides, regarding space complexity, it is easy to see that the size of $\mathcal{PK}'$, given in Proposition 2, is $\mathcal{O}(\mid \mathcal{PK} \mid + \mid \mathcal{S} \mid)$, where $\mid \mathcal{PK} \mid$ (resp. $\mid \mathcal{S} \mid$) is the size of the set $\mathcal{PK}$ (resp. of $\mathcal{S}$).

Example 6. Let us continue the previous example where we now show how to get the syntactic counterpart of adding the input information in the weighted knowledge base. Applying Proposition 2 gives us:

$$\begin{aligned} \mathcal{PK}' &= \mathcal{PK} \cup \{(p, \alpha)\} \cup \{((h \wedge \neg c) \vee (\neg h \wedge c), \beta)\} \\ &= \{(\neg p, 1), (\neg c, 1), (h, 1), (\neg p \vee h, 5), (p, \alpha), ((h \wedge \neg c) \vee (\neg h \wedge c), \beta)\}\,. \end{aligned}$$

The associated distribution of $\mathcal{PK}'$ is given in Table 4.

4.4. *Shifting down an OCF by* $\kappa(\xi_1 \wedge \cdots \wedge \xi_n)$

The last step provides the syntactic characterization of shifting down the weight of each interpretation $\kappa(\omega)$ by $\kappa(\xi_1 \wedge \cdots \wedge \xi_n)$. This shifting operation ensures that the resulted ordinal conditional function $\forall \omega, k'(\omega) = \kappa(\omega) - \kappa(\xi_1 \wedge \cdots \wedge \xi_n)$ is normalized (namely there will exist at least an interpretation with a weight equals to 0).

In this section, we consider a very slight extension of the definition of weighted knowledge bases. In the following, the use of negative weights is allowed. These negative weights can only be used when assigning degrees to the contradictory formula ($\bot$). These negative weights express a guaranteed reward. However weights associated with non-contradictory formulas $\phi_i \neq \bot$ remain positive. The other definitions, in particular Definition 1, also remain unchanged.

Proposition 3. *Let* $\mathcal{PK} = \{(\phi_i, \alpha_i) : i = 1, \ldots, n, \alpha_i \in \mathbb{R}^\star\}$ *be a weighted knowledge base. Let* $\mathcal{PK}' = \{(\phi_i, \alpha_i) : i = 1, \ldots, n, \alpha_i \in \mathbb{R}^\star\}$ *be the weighted base obtained in the previous step (Proposition 2), and let* $\kappa_{\mathcal{PK}'}$ *be its associated distribution. The syntactic counterpart of:*

$$\kappa_{\mathcal{PK}} \star \mathcal{S}(\omega) = \kappa_{\mathcal{PK}'}(\omega) - \kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n)$$

is:

$$\mathcal{PK} \star \mathcal{S} = \mathcal{PK}' \cup \{(\bot, -\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n))\}\,.$$

The proof is immediate by applying Definition 1. Indeed,

$$\begin{aligned}
\forall \omega, \kappa_{\mathcal{PK}\star\mathcal{S}} &= \begin{cases} 0 & \text{if } \forall (\phi_i, \alpha_i) \in \mathcal{PK}\star\mathcal{S}, \omega \models \phi_i \\ \sum\{\alpha_i : (\phi_i, \alpha_i) \in \mathcal{PK}\star\mathcal{S} \text{ and } \omega \not\models \phi_i\} & \text{otherwise} \end{cases} \\
&\quad + \begin{cases} 0 & \text{if } \omega \models \bot \ (\textit{which is impossible}) \\ -\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n) & \text{otherwise} \end{cases} \\
&= \kappa_{(\mathcal{PK}\star\mathcal{S})}(\omega) - \kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n) \\
&= (\kappa_{\mathcal{PK}}) \star \mathcal{S}(\omega)\,.
\end{aligned}$$

Clearly, the shifting operation is done in $\mathcal{O}(1)$ and obviously the size of resulted base is in $\mathcal{O}(\mid \mathcal{PK} \mid)$ (the result of addicting one propositional formula to the initial knowledge base).

Example 7. Let us illustrate the last step using Proposition 3. We have:

$$\begin{aligned}
\mathcal{PK} \star \mathcal{S} &= \mathcal{PK}' \cup \{(\bot, -\kappa_{\mathcal{PK}}(\xi_1 \wedge \cdots \wedge \xi_n))\} \\
&= 1\,.
\end{aligned}$$

The final revised weighted knowledge base is:

$$\mathcal{PK}\star\mathcal{S} = \{(\neg p, 1), (\neg c, 1), (h, 1), (\neg p \vee h, 5), (p, \alpha), ((h \wedge \neg c) \vee (\neg h \wedge c), \beta), (\bot, -1)\}$$

Table 5. The final ordinal conditional distribution associated with the weighted knowledge base resulting from revising $\mathcal{PK}$ with $\mathcal{S} = \{(\xi_1, \beta_1), \ldots, (\xi_n, \beta_n)\}$.

ω	p	c	h	$k_{\mathcal{PK}} \star \mathcal{S}(\omega)$
ω_0	0	0	1	α-1
ω_1	0	0	0	α+β
ω_2	0	1	1	α+β
ω_3	1	0	1	0
ω_4	0	1	0	1+α
ω_5	1	1	1	1+β
ω_6	1	0	0	6+β
ω_7	1	1	0	7

and its associated distribution is given in Table 5. We can clearly check that Table 5 is the same as the one of Table 3.

Propositions 1–3 provide the full characterization of the C-revision using weighted logic bases.

Note that the space complexity is linear with respect to the initial knowledge base for the three steps proposed in Propositions 1, 2 and 3. This is a major advantage comparing with a max-based encoding (instead of sum-based encoding) where the space complexity is exponential.[40]

The computational time complexity is also linear for steps given in Propositions 2 and 3. However computing $\kappa(\xi_1 \wedge \cdots \wedge \xi_n)$ (the step given in Proposition 1) needs a polynomial number of calls to a Max-SAT prover (an NP-Complete problem) with respect to the size of the knowledge base.

5. Concluding Discussions

Multiple iterated C-revision is a revision process that modifies an ordinal conditional function by taking into account a set of independent formulas. In this paper, multiple iterated C-revision has been encoded using weighted knowledge bases. This is done in three steps: i) computing the degree of the input in initial knowledge base $\kappa(\xi_1 \wedge ... \wedge \xi_n)$, ii) integrating the input information in initial knowledge base $\kappa(\omega) + \sum_{(\xi_i,\beta_i)\in\mathcal{S},\omega\not\models\xi_i} \beta_i$ and iii) shifting down an ordinal conditional function κ by a constant number. The shifting down operation is possible by introducing negative weights associated with a contradictory formula. This expresses a positive award associated with interpretations. A nice feature of the syntactic computation is space complexity which is linear with respect to the sizes of initial weighted knowledge base $\mathcal{PK}$ and the new information $\mathcal{S}$. Results of this paper have been defined in the framework of ordinal conditional functions. They can be easily rephrased in the context of product-based possibilistic logic thanks to the strong relations between ordinal conditional functions and product-based possibility theory.

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