EngD - INDEPENDENT STUDY

NUMERICAL OPTIMIZATION METHODS

Level 7:
Credit value: 20 credits (ECTS equivalent credit value 10)

PRE-REQUISITES AND CO-REQUISITES: None

AIMS

This unit provides Postgraduate Researchers (PGRs) with the concepts underlying commonly used numerical optimization methods. It teaches students how to choose the appropriate numerical method for a particular problem in image processing and computer graphics and to interpret the resulting output. Finally, students will be familiar with numerical optimization tools and computational software packages so that there will be no mathematic impediments or as little as possible for PGRs in reading research papers.

INTENDED LEARNING OUTCOMES

Having completed this unit the student will:

1) Display an *understanding* of the appropriate numerical methods for solving problems in an analytic fashion.
2) Demonstrate the ability to *critically assess* optimisation methods in solving numerical problems.
3) Be confident in assessment and selection of the appropriate solution for the applications.
4) Be able to effectively communicate technical areas of numerical analysis with peers, supervisors and others.

LEARNING AND TEACHING METHODS

This unit will employ a mix of lectures, workshops, online seminar and personal study.

ASSESMENT

This unit will be assessed by (1) the implementation of optimisation methods by coding; (2) a written report analysing key concepts and assessing performance of their implementation (Equivalent 2000 words).
A taken home coursework will be due by the end of each module.
(Alternatively, if you prefer to publications instead of the coursework, it is likely to negotiate your plan with tutor on a basis of case by case.)

INDICATIVE CONTENT

PGRs will negotiate a programme of study including 4 out of the 7 available modules. Credits will be awarded to PGRs who attend and pass 4 of these modules.

Delivery of each module will be via Fridays' lectures/workshops, followed by homework of independent learning.

MODULES

Module 1: Linear Programming Methods

In various optimization applications, there is a kind of problems whose objective functions can be expressed in a linear combination form, which is the Linear Programming Problem. In this module, we will study its standard form, basic solution, feasible solution and optimal solution, and then yield the simplex method through a computational example. Moreover, as the extension of linear programming, we will also study the Interior Point method and Semi-Definite Programming since they are more suitable for the large scale linear programming problems.

Module 2: Unconstrained Nonlinear Programming—Newton's Method

In optimization applications, Newton's method is one of the well-known methods. At the beginning of this module, we will review some basic concepts, such as, continuity and limits of the multivariable functions, gradient, Hessian matrix, Taylor expansion, quadratic form, convex optimization etc. Then, we will study the steepest descent method, Newton's method, Gaussian Newton method and Damped Newton methods. The latter is indeed a regularized method. These basic ideas have been widely applied in many optimization problems, such as CNN network training.

Module 3: Unconstrained Nonlinear Programming—Quasi Newton Methods and Conjugate Gradient Method

At the beginning of this module, we will revisit convex optimization and some related concepts, such as convex sets, convex functions, norms, nullspace, matrix decomposition and quadratic optimization problems. Then, we will study two kinds of the famous nonlinear optimization algorithms, Quasi-Newton methods and Conjugate Gradient methods. The former includes the DFP algorithm, BFGS algorithm and Limited memory-BFGS algorithm. The latter includes the usual conjugate gradient method and Newton-CG method. By numerical examples, students will be able to grasp the intrinsic linkage of these algorithms. Our study will focus on both algorithm analysis and numerical implementation.

Module 4: Constrained Nonlinear Programming—Lagrange Multipliers

Constrained nonlinear programming problems are usually encountered in our research. There is an intensive literature that addresses the theory and their implementations. Before starting this theme, we will have a revision of the basic calculus and linear algebra knowledge, such as, continuity and limits of the multivariable functions, derivatives and directional derivatives, mean value theorem, feasible sets, norms of vectors and matrices, nullspace and range, pseudo-inverse, conditioning and stability. Then, the module will introduce a few of famous methods, Lagrange multipliers, Augmented Lagrangian Method, Alternating direction method of multipliers. The latter (ADMM) is the recent focus in Big Data field.

Module 5: Nonlinear Least Squares Methods

Least squares methods have been widely applied to many fields, e.g. math, physics, chemistry and computer science. At the beginning of this module, we will briefly review the usual methods of solving a linear system, such as, Gaussian elimination, backward substitution, LU decomposition, Pivoting strategies, efficient implementation, Special matrices (symmetric positive definite, diagonally dominant, sparse), estimating errors and condition numbers, ill-conditioned problems. We will then introduce the linear and nonlinear least square methods separately. The former includes, over-determined systems, weighted least squares, moving least squares and constrained least squares. The latter includes, Guass-Newton method and Levenberg-Marquardt Algorithm that is regarded as the standard algorithm for nonlinear least squares problems. After that, we will move to the special issue, total least square problem.

Module 6: Quadratic Programming and Convex Optimization

This module refers to a special class of mathematical optimization problems, i.e. convex optimization, including least-squares and linear programming problems. It is well known that convex optimization arises in a variety of applications, and can be solved numerically very efficiently. Several related recent developments have stimulated new interest in the topic. The noteworthy is the recognition that interior-point methods, developed in the 1980s to solve linear programming problems, can be used to solve convex optimization problems as well. These new methods allow us to solve certain new classes of convex optimization problems, such as semidefinite programs and second-order cone programs, almost as easily as linear programs. We will revisit interior-point methods with a focus on semidefinite programming in our lectures since they have been widely employed in computer graphics and computer vision fields.

Module 7: Iterative method for Ax=b—Generalized Minimum RESidual method (GMRES)

In practice, we usually encounter large and sparse linear systems contaminated by noise. Generalized Minimum RESidual method (GMRES) is proposed to handle this challenge. At the beginning of this module, we will review some basic concepts of matrix computation. Then, we will study a few of the iterative methods, such as the Jacobian method, Gauss-Seidel method and Successive over-relaxation method. After that, we will further study GMRES by a computational example.

Module 8: Iterative methods—Preconditioning (optional)

In mathematics, preconditioning is a transformation procedure that conditions a given problem into a form that is more suitable for numerical solution. Preconditioning is typically related to reducing a conditional number of the problem. The preconditioned problem is then usually solved by an iterative method. At the beginning of this module, we will review the steepest descent method and conjugate gradient method, both iterative methods. Then we will focus on the Preconditioned Conjugate Gradient Algorithm.

Software: MatLab on windows

Further Reading

C.T. Kelley, Iterative Methods for Optimization, Frontiers in Applied Mathematics 18, SIAM 1995,

William H. Press, Saul A. Teukolsky, William T. Vetterling and Brian P. Flannery, Numerical Recipes in C, (2nd Edition), CAMBRIDGE UNIVERSITY PRESS, 1992, *Gene H. Golub and Charles F. Van Loan, Matrix Computations (Johns Hopkins Studies in Mathematical Sciences, 3rd Edition), The Johns Hopkins University Press, 1996,*

Richard L. Burden and J. Douglas Faires, Numerical methods, (9th Edition), Brooks-Cole, Cengage Learning, 2010,

Stephen Boyd, Lieven Vandenberghe, Convex Optimization, Cambridge University Press, 2004,

Jorge Nocedal and Stephen J. Wright, Numerical optimization, Springer series in operations research, 1999,